

Preface

In this PhD Thesis, we have studied several problems related to statistical properties of homogeneous, isotropic and turbulent flow of conducting fluid with direct numerical simulations (DNS) of equations of magnetohydrodynamics (MHD) and simplified shell models.

The Thesis begins with an introductory overview of several statistical characterisation of fluid turbulence and MHD turbulence. Chapter-1 discusses various challenges in turbulence in MHD context. This chapter also describes specific problems that are attempted in this Thesis.

The first problem, contained in Chapter 2, deals with dynamo action in a shell model for magnetohydrodynamic (MHD) turbulence. We have carried out systematic and high-resolution studies of dynamo action in a shell model over a wide range of the magnetic Prandtl number Pr_M and the magnetic Reynolds number Re_M . Our study suggests that it is natural to think of dynamo onset as a nonequilibrium, first-order phase transition between two different turbulent, but statistically steady, states. The ratio of the magnetic and kinetic energies is a convenient order parameter for this transition. By using this order parameter, we obtain the stability diagram (or nonequilibrium phase diagram) for dynamo formation in our MHD shell model in the (Pr_M^{-1}, Re_M) plane. The dynamo boundary, which separates dynamo and no-dynamo regions, appears to have a fractal character. We obtain hysteretic behavior of the order parameter across this boundary and suggestions of nucleation-type phenomena.

In Chapter 3 we present the results of our detailed pseudospectral direct numerical simulation (DNS) studies, with up to 1024^3 collocation points, of incompressible, magnetohydrodynamic (MHD) turbulence in three dimensions, without a mean magnetic field. Our study concentrates on the dependence of various statistical properties of both decaying and statistically steady MHD turbulence on the magnetic Prandtl number Pr_M over a large range, namely, $0.01 \leq Pr_M \leq 10$. We obtain data for a wide variety of statistical measures such as probability distribution functions (PDFs) of moduli of the vorticity and current density, the energy dissipation rates, and velocity and magnetic-field increments, energy and other spectra, velocity and magnetic-field structure functions, which we use to characterise intermittency, isosurfaces of quantities

such as the moduli of the vorticity and current, and joint PDFs such as those of fluid and magnetic dissipation rates. Our systematic study uncovers interesting results that have not been noted hitherto. In particular, we find a crossover from larger intermittency in the magnetic field than in the velocity field, at large Pr_M , to smaller intermittency in the magnetic field than in the velocity field, at low Pr_M . Furthermore, a comparison of our results for decaying MHD turbulence and its forced, statistically steady analogue suggests that we have strong universality in the sense that, for a fixed value of Pr_M , multiscaling exponent ratios agree, at least within our error bars, for both decaying and statistically steady homogeneous, isotropic MHD turbulence.

Chapter 4 is devoted to pseudospectral direct numerical simulation (DNS) studies of the three-dimensional magnetohydrodynamic (MHD) equations (3DRFMHD) stirred by a stochastic force with zero mean and a variance $\sim k^{-3}$, where k is the wavevector, for magnetic Prandtl numbers $\text{Pr}_M = 0.1, 1$, and 10 . We obtain velocity and magnetic-field structure functions and, from these, the multiscaling exponent ratios ζ_p/ζ_3 by using the extended self similarity (ESS) procedure. These exponent ratios lie within error bars of their counterparts for conventional three-dimensional MHD turbulence (3DMHD). We carry out a systematic comparison of the statistical properties of 3DMHD and 3DRFMHD turbulence by examining various probability distribution functions (PDFs), joint PDFs, and isosurfaces of quantities such as the moduli of the vorticity and the current density.

In Chapter 5 we present a study of the multiscaling of time-dependent velocity and magnetic-field structure functions in homogeneous, isotropic fluid turbulence. We first present a generalisation for magnetohydrodynamics of the formalism that has been developed for analogous studies of time-dependent structure functions in fluid turbulence. We then carry out a detailed numerical study of such time-dependent structure functions in a shell model for MHD turbulence. From this study we extract both equal-time and dynamic multiscaling exponents; however, we have not so far been able to come up with the MHD analogues of the linear bridge relations that relate equal-time and dynamic multiscaling exponents in fluid turbulence; indeed, it is not clear whether such bridge relations should exist for MHD turbulence.